DIRECT MEASUREMENT OF ISOTHERMAL FLOW STRESS OF METALS AT ELEVATED TEMPERATURES AND HIGH STRAIN RATES WITH APPLICATION TO Ta AND Ta–W ALLOYS

S. NEMAT-NASSER and J. B. ISAACS

Center of Excellence for Advanced Materials, Department of Applied Mechanics and Engineering Sciences, University of California, San Diego, La Jolla, CA 92093-0416, U.S.A.

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Abstract—A technique is developed for measuring the flow stress of metals over a broad range of strains, strain rates, and temperatures, in uniaxial compression. It utilizes a recent, enhanced version of the classical (Kolsky) compression split Hopkinson bar [1], in which a sample is subjected to a single stress pulse of a predefined profile, and then recovered without being subjected to any other additional loading. For the present application, the UCSD’s split Hopkinson bar is further enhanced by the addition of a new mechanism by means of which the incident and transmission bars of the split Hopkinson construction are moved into a constant-temperature furnace containing the sample, and gently brought into contact with the sample, as the elastic stress pulse reaches and loads the sample. The sample's temperature is measured by thermocouples which also hold the sample in the furnace. Since straining at high strain rates increases the sample’s temperature, the sample is allowed to attain the furnace temperature after each controlled incremental loading, and then is reloaded, using the same stress pulse and strain rate. The technique also allows for checking any recoveries that may occur during unloading and reloading. Using several samples of the same material and testing them at the same strain rate and temperature, but different incremental strains, an accurate estimate of the material’s isothermal flow stress can be obtained. Additionally, the modified Hopkinson technique allows the direct measurement of the change in the (high-strain-rate) flow stress with a change of the strain rate, while the strain and temperature are kept constant, i.e., the strain rate can be increased or decreased during the high-strain-rate test. The technique is applied to obtain both quasi-isothermal and adiabatic flow stresses of tantalum (Ta) and a tantalum–tungsten (Ta–W) alloy at elevated temperatures. These experimental results show the flow stress of these materials to be controlled by a simple long-range plastic-strain-dependent barrier, and a short-range thermally activated Peierls mechanism. For tantalum, a model which fits the experimental data over strains from a few to over 100%, strain rates from quasi-static to 40,000/s, and temperatures from −200 to 1000°C, is presented and discussed.

1. INTRODUCTION

The compression split Hopkinson bar was developed by Kolsky [2], based on the original concept of John Hopkinson [3] and his son, Bertram Hopkinson [4, 5], who, respectively, sought to directly measure the dynamic strength of a wire in uniaxial tension and the stress profile transmitted into an elastic bar by the impact of a projectile. Kolsky sandwiched a specimen between two Hopkinson bars and developed a technique to directly measure the stress-strain response of the specimen subjected to uniaxial stress at an essentially constant strain rate. To measure the force transmitted through the specimen (the stress) and the relative displacement of the specimen-end of the incident and transmission bars (the strain), he used an electronic technique earlier developed by Davis [6]. Kolsky’s construction is known as the Kolsky bar. It has been modified to measure uniaxial flow stress in tension, as well as the flow stress in torsion; see Refs [7–12]. While the general technique allows direct measurement of the one-dimensional stress-strain response of a sample, it is not possible by this method to relate the microstructural changes to the associated loading history, since the specimen is additionally subjected to repeated loading by the elastic waves which reflect off the free ends of the bars.

Recently, Nemat-Nasser et al. [13] have suitably modified the Kolsky compression, as well as tension techniques to allow subjecting the sample to a single stress pulse of predetermined profile and then recovering the sample without it having been subjected to any additional loading. In the compression test, this is accomplished through a construction which either traps or renders tensile all reflected pulses which travel toward the sample, once the initial compressive loading of the sample is completed. In the tension test the sample is subjected to a single tensile pulse by trapping both the transmitted and the reflected pulses emanating from the sample-bar interfaces.
In the present work, we describe a further enhancement of the compression recovery Hopkinson technique (henceforth referred to as the UCSD technique) which allows measurement of the isothermal flow stress of certain metals over a broad range of temperatures and strain rates. At room temperature, the UCSD technique is used to strain the sample incrementally, allowing (upon the completion of each increment) the sample to cool to room temperature, and then reloading it at the same strain rate [13]. A certain amount of recovery may occur during the unloading and reloading process for certain materials. This can be assessed by employing overlapping strain increments, as will be detailed later on herein.

For high-temperature tests a furnace is employed to preheat the specimen while keeping the transmission and incident bars outside of the furnace. These bars are then automatically brought into gentle contact with the specimen, just before the stress pulse reaches the specimen-end of the incident bar. Incremental straining is used to measure the strain softening produced by the temperature rise due to the corresponding plastic deformation of the sample. Here again, the effect of any recovery on the flow stress that may occur during unloading and reloading the sample, can be assessed by overlapping the incremental straining of similar samples of the material. The basic technique is discussed and the results are illustrated, using tantalum and tantalum-tungsten alloys which have high flow stress and are highly temperature-sensitive. The flow stresses of pure tantalum and a tantalum-tungsten alloy are obtained at several strain rates and initial temperatures, leading to quasi-isothermal and adiabatic stress-strain curves. An interesting observation is the strong workhardening of pure tantalum and tantalum-tungsten alloys over a broad range of temperatures.

A close examination of the low-temperature isothermal and high-temperature adiabatic flow stress of tantalum and tantalum-tungsten alloys reveals that the resistance to the dislocation motion over the temperature range, from room to 1000 K, and strain rates from 1000 to 40 000 s⁻¹, is essentially due to long-range obstacles such as grain boundaries and farfield dislocation forests, and short-range thermally-activated Peierls barriers. For b.c.c. crystals, this has been pointed out by many investigators, for example Refs [14–22]. However, the lack of definitive experimental results precluded the formulation of a satisfactory model which fits the experimental data over a broad range of strains, strain rates, and temperatures. Various models, e.g. those of Johnson and Cook [23], Zerilli and Armstrong [22] and the Mechanical Threshold Stress (MTS) [24], have recently been compared by Chen et al. [25] to experimental results, over a broad range of temperatures and strain rates, but the strains were limited to less than 30%. Better correlations with experimental data are reported for the MTS model.

In the present work it is shown that a combination of the long-range, plastic strain-dependent barrier and a short-range temperature- and strain rate-dependent barrier provides a simple and effective description for the flow stress of tantalum and tantalum-tungsten alloys, over a broad range of strains, strain rates, and temperatures.

In Section 2, UCSD's enhanced recovery Hopkinson technique is described and compared with the classical Kolsky bar. In Section 3, the procedure for obtaining the isothermal flow stress at high strain rates and high temperatures, is discussed and experimental results for a tantalum-10% tungsten alloy and for tantalum are presented. In Section 4, a flow stress model for tantalum is developed and the results are compared with experimental data.

2. THE UCSD HOPKINSON COMPRESSION TECHNIQUE AND ITS ENHANCEMENT

The details of the UCSD Hopkinson recovery techniques are given in Ref. [1]. Before detailing the enhancement of the compression technique, a brief review of this method is given.

2.1. The classical Kolsky and the UCSD method

Kolsky's construction consists of an incident bar, a transmission bar (with the sample sandwiched between them) and a striker bar which imparts an elastic compression stress pulse into the incident bar; see Fig. 1. The pulse travels toward the sample, loading the sample in compression. A fraction (usually small) of the momentum carried by the pulse is transmitted through the sample into the transmission bar, while the remaining part of the stress pulse is reflected off the sample as a tensile pulse traveling back toward the striker-end of the incident bar. Strain gauges placed at the midpoints of the two bars are used to measure the transmitted pulse (stress) and the reflected pulse (strain rate) in the sample. From these the stress-strain curve of the sample is developed at various, essentially constant, strain rates.

While the transmitted compression pulse is reflected off the free end of the transmission bar as

![Fig. 1. Classical compression split Hopkinson bar (Kolsky bar) the sample is subjected to repeated loading by the reflected pulses.](image-url)
tension, the tension pulse reflected off the sample back into the incident bar will travel to the free end of this bar and then will return back to the sample as compression.

To trap the tension pulse (which reflects off the sample into the incident bar) in the UCSD technique, a transfer flange and an incident tube are added to the striker-end of the incident bar, as shown in Fig. 2. The impedance of the incident tube is the same as that of the incident bar. Once the tension pulse is reflected off the free end of the transfer flange as compression, it loads in compression the incident tube in contact with the transfer flange, thereby transmitting the entire momentum into this tube. This compression then reflects off the free end of the tube as tension, and is trapped in the tube, since the interface between the tube and the flange does not transmit tension. In the experiment, a precision gap is initially placed between the incident tube and the transfer flange, such that the gap is closed, once the entire initial compressive stress pulse is transmitted by the striker bar into the incident bar. In a well-conducted experiment, the entire reflected pulse is trapped, so that, subsequently, essentially no signal is detected by the strain gauge on the incident bar.

The transfer flange and the incident tube construction can also be used to change (increase or decrease) the strain rate in the middle of an experiment.

It should be noted that a compression recovery split Hopkinson bar can be affected by making the transmission bar suitably shorter than the incident bar. This, however, precludes the capability to change the strain rate during the test. In addition, the relative lengths of the two bars must be adjusted with care for each experiment, which makes the technique time-consuming, expensive, and not necessarily always successful. For example, when long striker bars are used, the tensile pulse which reflects off the free end of the transmission bar may reach the sample while the sample is still under compression. The momentum-trap construction, on the other hand, provides a permanent and effective recovery capability, alleviating any such difficulties. Moreover, the UCSD design for trapping the tension pulse can be effectively used in the tension recovery Hopkinson technique, see Ref [1].

2.2. The enhanced UCSD technique

For high strain-rate tests at elevated temperatures, it is necessary to heat the sample to the required temperature, while keeping the incident and transmission bars at suitably low temperatures. If the bars are in contact with the specimen within the furnace, their temperature will increase, having a variable distribution along the bars. This affects the bars' elastic properties and hence, the stress pulses. Moreover, the bars being good heat conductors (usually of maraging steel), the considerable heat loss that occurs, makes controlling the experiment difficult.

To avoid these and related difficulties, the bars are kept outside the range of the heating unit in the furnace, while keeping the specimen at the centre of the furnace. The bars are then brought into contact with the specimen, microseconds before the stress pulse reaches the end of the incident bar. This is accomplished by two bar movers which are activated by the same gas gun that propels the striker bar toward the incident bar. Figure 3(a) illustrates the experimental setup just before the gun is fired, and Fig. 3(b) shows the configuration after the bar movers have brought the incident and transmission bars in contact with the sample. The motion of the bar movers is controlled by the area of its piston and the gas pressure in the breach of the gas gun.

In the UCSD design, the bars are brought into contact with the specimen a few microseconds before the sample is loaded by the elastic pulse travelling in the incident bar. Once the sample is loaded, the bars move out and the sample is recovered without having been subjected to any stresses other than the initial one.

Figure 4(a) shows the furnace and the ends of the incident and transmission bars. Figure 4(b) shows the relative position of the sample held by thermocouples, and the ends of the incident and transmission bars; this is the configuration maintained in the furnace just before firing the gas gun. The sample is attached by suitable wires to a sleeve, which is a thin tube.
Fig. 4(a, b).
Fig. 4. (a) Furnace over the ends of the incident and transmission bars; (b) position of the sample when held by thermocouples; (c) furnace and the sample attached to the sleeve; (d) bar movers attached to the breech of the gas gun of the Hopkinson construction.
3. ISOThERMAL FLOW STRESS AT HIGH STRAIN RATES

To construct an isothermal flow stress at a high strain rate, the sample is heated to the required temperature in the furnace attached to the recovery Hopkinson bar, and then loaded incrementally. After the application of each load increment, the sample is unloaded without being subjected to any additional stress pulses. The sample is then allowed to return to the furnace temperature before the application of the next strain increment.

Since the unloading, the cooling of the sample to its initial temperature, and the reloading, may affect the microstructure and hence the thermomechanical properties of the material, it is necessary to check this in each case. To this end, a sample which has been loaded, unloaded, and cooled to its initial temperature, may then be reheated to the temperature that it had just prior to its unloading. If there are no substantial changes in the microstructure that affect the flow-stress properties, then the flow stress, upon reloading, should follow the previous stress-strain curve. For a tantalum-10% tungsten (Ta-10%W) alloy, this is illustrated in Fig. 5.

In Fig. 5, the dashed curve is obtained by loading the sample at about 5700 s⁻¹ strain rate, to a true strain of about 75%. This curve represents the (essentially) adiabatic, true stress-true strain relation for the material. The solid curve (1) is obtained by taking another sample of the same material, loading it to about 23% strain at the same strain rate, and then unloading it. The fact that curve (1) follows closely the dashed curve is an indication of the good quality of the test and the fact that the two samples represent the same material with essentially the same initial conditions.

Now, to check if any recovery has occurred due to unloading and cooling, we have calculated the total plastic work and, assuming that essentially all of it has been used to heat the sample, have estimated that the sample temperature just prior to unloading must have been about 110°C more than its initial room temperature. Thus, the sample which had then attained room temperature after unloading, was first heated to about 135°C in the furnace, and loaded at the same strain rate. This has produced the second solid curve, marked (2), shown in Fig. 5. This also follows closely the dashed curve. Two important conclusions are drawn from these results:

1. if there was any recovery, it did not affect the flow stress noticeably;
2. essentially all the plastic work had been converted into heat.

As an application of the UCSD Hopkinson recovery technique, consider measurement of the high strain rate isothermal flow stress of the Ta-10%W alloy, at room temperature (25°C) and at 325°C; see Fig. 6(a) and (b) respectively. The room-temperature results have been reported by [13], but the results for the 325°C initial temperature are new.

For the sake of completeness, Fig. 6(a) also includes the results of isothermal true stress-true-strain curves obtained at 10⁻³ s⁻¹ (curve 1) and 1 s⁻¹ (curve 2), respectively. Curve 3 in Fig. 6(a) is obtained by testing a sample at a 5700 s⁻¹ strain rate to the indicated strain. At such high strain rates, the deformation is basically adiabatic. Curves 4, 5 and 6 in Fig. 6(a) are obtained by testing a sample incrementally at a 5700 s⁻¹ strain rate, each time allowing the sample to cool down to room temperature (25°C) before reloading it. Hence, curve 7 in Fig. 6(a) may be regarded as the room temperature quasi-isothermal true stress-true strain relation at a 5700 s⁻¹ strain rate. Note that the isothermal curves (1), (2) and (7) are essentially parallel.

With a similar approach, using an attached furnace, the results given in Fig. 6(b) for a 325°C initial temperature have been obtained. Figure 6(b) compares the flow stresses of this metal at 25°C and 325°C initial temperatures. The quasi-isothermal curves are almost parallel, showing essentially the same workhardening for the Ta-10%W alloy. Comparing these results with those of Fig. 6(a), it is seen that the isothermal workhardening is independent of the temperature for this material. Hence, the workhardening due to plastic straining is not coupled with either temperature or strain rate. This is an important property, since, then, the flow stress can be divided into an athermal part and a thermally-activated part, where the athermal part
Fig. 6. (a) The true stress-true strain relations for Ta-10% W at 25°C; curves (1), (2) and (7) are isothermal relations at strain rates of $10^{-7}$, 1 and 5700 s$^{-1}$; curve (5) is adiabatic at 5700 s$^{-1}$ strain rate. (b) Effect of initial temperature on true stress-true strain relations for Ta-10% W; curves (1), (2), (3) and (4) are for 25°C and (5), (6), (7) and (8) are for 325°C initial temperatures; strain rate is 5700 s$^{-1}$.

4. PHYSICALLY BASED MODEL FOR TANTALUM

For crystals which deform plastically, essentially through dislocation motion and accumulation, models have been developed based on the notion of thermally activated dislocation kinetics, for moderate strain rates (say, less than $10^8$ s$^{-1}$), and the notion of a dislocation-drag mechanism for deformations at greater strain rates: see for example Refs [2, 17–19, 21, 24, 26–28]. The motion of dislocations through the crystals of a polycrystal is a complex phenomenon with various features which may not lend themselves to simple mathematical models. In the present work the concept of the motion of dislocations and the barriers they must overcome in this process, is used as an underlying motivation to obtain general expressions which include a number of free constitutive parameters. These parameters are then evaluated by direct comparison with experimental data. It turns out that at least some of the parameters have clear physical interpretations. Their order of magnitude is, therefore, estimated directly from the microstructure of the material. Furthermore, the effectiveness of the resulting constitutive model is tested, using independent experiments which are not employed in the estimate of the constitutive model.

Before accepting curve (7) in Fig. 6(a) as the high strain-rate isothermal flow stress of the Ta-10% W material, which was used in this experiment, one must establish whether or not a uniform stress state is attained in the sample, prior to yielding in each incremental straining. Since the sample is about 0.15 in long, the duration for the elastic wave to travel the length of the sample is about 1.2 μs. It is easy to show that the stress state in the sample is essentially uniform after 5 μs. The time required by the applied stress pulse to bring the samples to the yield state is greater than 10 μs. Hence, a uniform stress state in the sample is attained long before the material begins to yield at each incremental loading.

Figure 7(a) gives the adiabatic flow stress of pure tantalum tested at a 5000 s$^{-1}$ strain rate and at the indicated initial temperatures. The quasi-isothermal curves for 25, 125, 225, 325°C initial temperatures are shown in Fig. 7(b). Similarly to the results for the Ta-10% W alloy, shown in Fig. 6(b), the isothermal flow-stress curves in Fig. 7(b) are also essentially parallel to each other.
parameters. It turns out that the same constitutive relation fits a large amount of independently obtained published data.

4.1. Data analysis

As a starting point, the measured flow stress, shown by the curves in Fig. 7(a), (b), are replotted in Fig. 8(a), as stress vs absolute temperature, for indicated plastic strains. This figure also includes estimates obtained, using the adiabatic flow stress curves in Fig. 7(a), for high temperatures, e.g. 998 and 798K. Since in these cases the temperature is high and the flow stress is rather low, the change in the temperature due to the plastic work is small relative to the total temperature and can be estimated directly from the corresponding stress-strain curves, without introducing errors of any significance to the analysis. These adjusted points are also included in Fig. 8(a).

4.2. Effect of long-range barriers

Examination of the trend in Fig. 8(a) suggests that the flow stress becomes essentially independent of the temperature, close to 1000K. These high-temperature limiting values of the flow stress are plotted in Fig. 8(b), as stress versus the corresponding plastic strain. The points in Fig. 8(b) nicely fit a simple power law,

$$\tau = \tau_a \gamma^n, \quad \tau_a = 473 \text{ MPa}, \quad n = \frac{1}{3} \quad (1)$$

This is taken as the athermal part of the flow stress for this material, writing the total flow stress as the sum of the athermal and a thermally-activated part, $\tau^*$, as follows:

$$\tau = \tau_a + \tau^* \quad (2)$$

Notation. We use $\tau$ and $\gamma$ for stress and the corresponding strain both in the uniaxial compression tests and in the modelling of the flow stress in terms of the strain, strain rate, and temperature. The flow stress and strain relate to the uniaxial stress and strain by the Taylor factor which we assume to be included in the definition of $\tau$ and $\gamma$, when these are used to represent the flow stress and the corresponding strain; see also comments in Section 4.8.

4.3. Effect of short-range barriers

For b.c.c. metals, the lattice itself provides the short-range barrier (the Peierls barrier) to the motion of dislocations. Generally, a double kink is formed with the assistance of thermal vibrations, and the kinks then move sideways, leading to the advancement of the dislocation line. In this process, the kinks may be pinned down by other defects or alloying elements, thereby increasing the resistance to plastic flow. For Ta-W alloys, the tungsten atoms may pin down the dislocations in their motion. Hence, the flow stress for Ta-10%W is considerably higher than that for pure tantalum; compare Figs 6 and 7. In what follows, the stress $\tau^*$ is assumed to correspond solely to the Peierls resistance to the dislocation motion. Note that $\tau_a$ and $n$, in equation (1), should be viewed as free constitutive parameters which must be empirically estimated for each tantalum material. Their values will depend on the texture and grain size and the shape distributions of the material; see Subsection 4.6.

Based on equations (1) and (2), the results of Fig. 8(a) are now replotted in Fig. 8(c) as $\tau^* = \tau - \tau_a$ vs the temperature, $T$. As is seen, all the experimental points collapse into basically a single curve which may be represented by the
following mathematical description:

$$\tau^* = 1100(1 - (0.001T)^{1.2})^{1/2}$$  \(3\)

This equation is shown by a solid curve in Fig. 8(c). The choice of the exponents in equation (3) are discussed in the next subsection. The coefficient 0.001, of the temperature, \(T\), is selected by noting that the influence of thermal activation disappears once the temperature reaches a critical value of about \(T_c = 1000K\).

It is shown subsequently that equation (3) is a special form of the following more general expression:

$$\tau^* = \left[1 - \left(-\frac{k}{G_c} T \ln \left(\frac{j'}{j'}\right)\right)^{1/p}\right]^{1/2}$$  \(4\)

where \(j\) is the strain rate, \(k\) is the Boltzmann constant, \(\tau\) is the threshold stress of the Peierls barrier to the dislocation motion, and \(G_c\) is the corresponding energy. The reference strain rate is given in terms of the density of the mobile dislocations, \(\rho_m\), and the magnitude of the Burgers vector, \(b\), by

$$j = b^2 \rho_m \nu$$  \(5\)

as shown below; here, \(\nu\) is an “attempt frequency”.

4.4. Comparison with data used for modelling

Figure 9 compares the adiabatic flow stress given by

$$\tau = 473\gamma^{1/3} + 1100(1 - (0.001T)^{1.2})^{1/2}$$  \(6\)

with the experimental data. The temperature, \(T\), is calculated by assuming that all the plastic work is converted into heat. As pointed out before, our experiments show this to be essentially the case for large plastic strains. Initially, however, a portion of the plastic work is stored in the material in the form of the elastic energy of the dislocations. This is quickly saturated, and, as seen from Fig. 5, it does not seem to have any significance in the present case.

Hence, the temperature is calculated incrementally from

$$\rho_c G_c \frac{dT}{d\gamma} = \tau(\gamma, T)$$  \(7\)

with \(T = T_i\) at \(\gamma = 0\), where \(\tau = \tau(\gamma, T)\) is defined by equation (6), \(T_i\) is the initial temperature, \(\rho_c\) is the mass density, and \(c\) is the heat capacity at constant volume. Since the flow stress varies gently with the plastic strain, the integration of equation (7) can be performed using the mean-value theorem or even the simple Euler method, without introducing any noticeable errors.

4.5. Temperature- and strain rate-dependent part of flow stress, \(\gamma^*\)

Define the plastic strain rate in terms of the magnitude of the Burgers vector, \(b\), the density of the mobile dislocations, \(\rho_m\), and the average dislocation velocity, \(\nu\), as

$$\dot{\gamma} = b^2 \rho_m \nu$$  \(8\)

As has been discussed by many authors (e.g. Ref. [19]), set

$$\nu = d \omega_0 \exp \left[-\frac{\Delta G}{kT}\right]$$  \(9\)

where \(d\) is the average distance the dislocation moves between the barriers, \(\omega_0\) is the attempt frequency, and \(k\) is the Boltzmann constant. Kocks has estimated \(\omega_0\) to be about 10^1 s^{-1}. The distance, \(d\), for the Peierls barriers will be taken to be equal to the Burgers vector, \(b\). As suggested by Kocks et al. [19], the profile of the Peierls barrier, defining the energy increment, may be estimated by,

$$\Delta G = G_c \left[1 - \left(\frac{T}{T_i}\right)\right]^{p}$$  \(10\)

where \(\tau\) is the threshold stress at which the dislocation can overcome the barrier without the assistance of thermal activation; \(G_c\) is the total energy barrier measured per unit barrier, and \(p\) and \(q\) are constants, 0 < \(p\) < 1, and 1 < \(q\) < 2. We have found that the fitting of the data is not very sensitive to the choice of \(p\) and \(q\). Indeed, even a linear relation, although not very good, could be considered adequate. However, excellent fit is obtained, with \(p = 2/3\) and \(q = 2\). These are the values used in equations (3) and (6). Indeed, it is suggested by Ono [29], as well as Kocks et al. [19], that these values of \(p\) and \(q\) provide a sufficiently accurate description of the profile of the energy barrier in most cases.

To obtain equation (4), first substitute from equation (9) into equation (8) arriving at,

$$\dot{\gamma} = \gamma \nu \exp \left[-\frac{\Delta G}{kT}\right]$$  \(11\)

where \(\dot{\gamma} = b^2 \rho_m \omega_0\). Then solve for \(\Delta G\), and use equation (10), arriving at equation (4). The final constitutive relation is obtained by substituting equations (1) and (4) into equation (2).

$$\tau = \tau_o \gamma^n + \dot{\tau} \left[1 - \left(-\frac{k}{G_c} T \ln \left(\frac{j'}{j'}\right)\right)^{1/p}\right]^{1/2}$$  \(12\)

Fig. 9. Comparison of the adiabatic flow stress for Ta at a 5000 s^{-1} strain rate with model predictions.
Since
\[ \ln \frac{\dot{\gamma}_i}{\dot{\gamma}_j} = \ln \frac{\dot{\gamma}_i}{\dot{\gamma}_j} + \ln \frac{\dot{\gamma}_i}{\dot{\gamma}_j}. \]

Equation (12) can be modified to include the effect of a change in the strain rate (from \( \dot{\gamma}_i \) to \( \dot{\gamma}_j \)) on the flow stress, as follows:
\[
\tau = \tau_0 \dot{\gamma}^n + \xi \left\{ 1 - \left[ -\frac{k}{G_0} T \left( \ln \frac{\dot{\gamma}_j}{\dot{\gamma}_i} + \ln \frac{\dot{\gamma}_i}{\dot{\gamma}_j} \right) \right] \right\}^{\frac{1}{1-n}}
\]

Once the flow stress at a given strain rate, say, \( \dot{\gamma}_i \), is obtained, then the flow stress at any other strain rate, say, \( \dot{\gamma}_j \), is given by simply modifying the strain-rate term, as indicated.

4.6. Discussion of equation (12)
In the present work, the orientation (or Taylor) factor has been absorbed into the definition of stress, \( \tau \), and plastic strain, \( \gamma \). For the experiments, we have prepared samples from tantalum disks which have been produced by rolling consecutively in two orthogonal directions, initial ingots. The disks have different directionally-dependent flow stresses. This may be in part due to the presence of directionally-dependent crystal orientations (texture), and in part due to the directionally-dependent distribution of the grain boundaries and the dislocation forests. These differences may lead to different values for \( \tau_0 \), \( n \), and \( \xi \), depending on the orientation of the sample relative to the rolling directions. The other constitutive parameters, namely \( k/G_0 \) and \( \dot{\gamma}_j \), however, are not expected to be significantly affected by the distribution of the grain shapes and the orientation of the crystals within the sample, although all the free parameters in equation (12) may have to be checked against experiments. Hence, in applying equation (12) to various tantalum plates or rods which may be obtained by rolling or forging, at least the constitutive parameters \( \tau_0 \) and \( \xi \) must be estimated by direct comparison with the experiments. In the presence of any texture, at a minimum, an orientation factor (Taylor factor) must be incorporated into equation (12). As discussed in the remaining subsections, all the experimental data which have been available to us, are modelled by equation (12) without any change or, at most, with only a small change in the values of \( \tau_0 \) and \( \xi \).

In equation (12), \( G_0 \) is the energy of the Peierls barrier per atom, and will be taken as \( \text{eV/atom} \) for the present application to tantalum. The reference strain rate, \( \dot{\gamma}_0 \), on the other hand, must be estimated by direct comparison with experiment. An order-of-magnitude estimate is obtained by noting that \( d = b \approx 3.31 \text{Å}, \rho_0 \approx 10^3 \text{s}^{-1} \), and \( \rho_0 \approx 10^8 \text{m}^{-2} \), leading to \( \dot{\gamma}_0 \approx 10^{18} \text{s}^{-1} \). From equation (3) and with \( G_0 = \text{eV/atom} \), it follows that
\[
k/G_0 = 8.62 \times 10^{-4} \text{K}^{-1}, \quad \dot{\gamma}_0 = 5.46 \times 10^8 \text{ s}^{-1} \quad (13)
\]

Note that in equation (3), 0.001 is given by \( T_e^{-1} \); see Fig. 8(c) and equation (14) below.

Finally, note that, in equation (12), the second term on the right-hand side, i.e. the thermal component of the flow stress, is non-negative, and should be set equal to zero when the temperature exceeds a corresponding critical value which is strain rate-dependent, and which is given by
\[
T_e = \left( \frac{k}{G_0 \ln \frac{\dot{\gamma}_j}{\dot{\gamma}_i}} \right)^{-1} \quad (14)
\]

For \( \dot{\gamma} = 10^{-3}, 10^{-1}, 1000, 5000, \) and \( 40000 \text{s}^{-1} \), this gives, \( T_e \approx 340, 520, 880, 1000, \) and \( 1220 \text{K} \), respectively.

With the constitutive parameters defined by equation (12), we now seek to check if equation (12) can be used to fit other independently obtained data for pure tantalum, with minimal change in the free parameters. To this end, equation (12) is written as
\[
\tau = \tau_0 \dot{\gamma}^n + \xi \left\{ 1 - \left[ 0.001 \times \left( 1 - 0.0862 \ln \frac{\dot{\gamma}_j}{5000} \right) \right]^{\frac{1}{1-n}} \right\}
\]

As discussed in Subsection 4.8, at a minimum, it is expected that the parameters \( \tau_0 \) and \( \xi \) must be adjusted even for samples obtained from the same tantalum plate, but from different locations or at different directions. Hence, in the following subsection, \( \tau_0 \) and \( \xi \) will be adjusted so that, at least one data point fits the prediction, and then it is checked to see how the remaining data are predicted. However, for the first set of data, it is found that good correlation is obtained without any change in the parameters.

4.7. Comparison with other experimental results
Vecchio [30] has published high-strain, high strain-rate data on tantalum. The experiments were performed by the second author (Jon Isaacs) of the present paper on tantalum with essentially similar properties as those of the material used to obtain the data shown in Fig. 7(a),(b). Since these experiments were completed before the present model was developed, and since the strain rates used were \( \dot{\gamma} = 10000, 18000, \) and \( 40000 \text{s}^{-1} \), with strains exceeding 100%, the data can provide a good check for the range of applicability of the present model.

Figure 10(a) compares the results obtained from equation (15), with the experimental data, where, for the model, the temperature \( T \) is calculated incrementally using equation (7). As is seen from this figure, no change is made in the parameters; i.e. the same \( \tau_0 = 473 \text{ MPa} \) and \( \xi = 1100 \text{ MPa} \) are used.
Equation (15) also fits the data recently reported by Chen et al. [25] for pure tantalum (Fig. 12), with an adjustment in the value of $r_0$; i.e. with $r_0 = 500$ MPa, and all other parameters unchanged. While the plastic strains were limited to less than 30%, the test temperatures and strain rates used by these authors varied from 77 to 1300K and $10^{-1}$ to $3900$ s$^{-1}$ respectively.

In Fig. 12, the three lowest curves correspond to 600, 800 and 1000°C (i.e. about 900, 1100, and 1300K) which are all above the critical temperature, $T_c$, for the associated strain rates of 2200, 3900 and $3000$ s$^{-1}$, as can be checked from equation (14). Hence, these three curves which are, for all practical purposes, essentially the same, should correspond to the athermal part of the flow stress, $\tau = \tau_0 = \tau_0^*$. Using $\tau_0 = 500$ and $n = 1/5$, the prediction of this equation for $\gamma = 0.025$, 0.05, 0.1, 0.15 and 0.2 is shown in Fig. 12, by $\Delta$, for the lowest three curves.

For lower temperatures, we have used equation (15) with $r_0 = 500$ MPa and $\tau = 1100$ MPa, to calculate the flow stress for the other curves in Fig. 12. These are also shown by various symbols in Fig. 12. As is seen, the predictions are reasonable. Although one may obtain better correlations by adjusting the free parameters in the general expression, equation (12), e.g. also $\tau$ and $\tau_0$, we felt this is not warranted, considering the limited data, and the lack of details as to the nature of the experiments.

Chen et al. [25] also report data for another tantalum which they refer to as Ta-bar. This material shows a lower flow stress than the tantalum which they identify by Ta-DD; see Fig. 12 above. However, for Ta-bar, quasi-static data are shown with prestraining of $\gamma_0 = 0.42, 0.95$ and 1.85, although Chen et al. [25] explain that “the reproducibility in the strain-stress response is diminished after pre-straining” which they comment to be due to the nonhomogeneities in their initial Ta-bar material.
Nevertheless, it may be useful to see if equation (15) with only adjustments in \( \tau_0 \) and \( \dot{\gamma} \), say, due to pre-existing crystal shape and orientation distributions, can fit within the range of the data shown by these authors.

To this end, in the equation given in Fig. 12, the parameters \( \tau_0 \) and \( \dot{\gamma} \) are proportionally adjusted to \( \tau_0 = 360 \) MPa and \( \dot{\gamma} = 790 \) MPa, so that at \( \gamma = 0.15 \), the flow stress \( \tau = 300 \) MPa is obtained for \( T_0 = 25^\circ \text{C} \) and \( \dot{\gamma} = 10^{-3} \text{ s}^{-1} \). Equation (15) with \( \tau_0 = 360 \) MPa and \( \dot{\gamma} = 790 \) MPa fits perfectly the low-strain data, as seen in Fig. 13. It also produces data which fit the lowest curves shown by Chen et al. [25] remarkably well, at \( \gamma = 0.42, 0.95, \) and 1.85. The predicted stresses are \( \tau = 356, 410, \) and 460 MPa, respectively.

A more systematic approach, however, is to consider the data given by Chen et al. [25] in their Fig. 2 for \( T_0 = 600^\circ \text{C} \) and \( \dot{\gamma} = 3200 \) s\(^{-1}\). Since the temperature then exceeds \( T_c \), the corresponding curve defines the athermal flow stress \( \tau = \tau_0 \gamma^{\frac{1}{3}} \). This yields \( \tau_0 \approx 390 \) MPa; see Fig. 14, where \( \tau = 390 \gamma^{\frac{1}{3}} \) is shown by open circles. Then, with the aid of one other data point, say, for \( T_0 = 200^\circ \text{C} \) and \( \dot{\gamma} = 3100 \) s\(^{-1}\), we have obtained \( \dot{\gamma} = 1300 \) MPa, with all other parameters unchanged. Using these values for \( \tau_0 \) and \( \dot{\gamma} \) in equation (15), we obtain the flow stresses which fit quite well the curves in Fig. 2 of Ref. [25] (see Fig. 14). Not shown in Fig. 14 are our predictions for \( \gamma = 0.001 \) and 0.1 s\(^{-1}\). They are, respectively, 275, 303, 335, 355 and 371 MPa at \( \gamma = 0.025, 0.05, 0.10, 0.15 \) and 0.20 for \( \gamma = 0.001 \) s\(^{-1}\). Similarly, we have 340, 368, 400, 421 and 437 MPa at the same strains for \( \gamma = 0.1 \) s\(^{-1}\). We have also calculated the stresses for \( \gamma = 0.42, 0.95, \) and 1.85 of Chen et al.'s Fig. 14, as 416, 475 and 530 MPa, respectively, but the low-strain, quasi-static (10\(^{-3}\) s\(^{-1}\)) data now are not as accurately reproduced. Comparison with other quasi-static data which we have obtained at various temperatures, seems to suggest that equation (12) fits the dynamic flow stresses better than the quasi-static ones. So far we have noted considerable scatter in our quasi-static data for samples from the same plate, whereas the dynamic data show better reproducibility.

Finally, Chen et al. report data on another low-oxygen tantalum, referred to as Ta-A. The tests are for a 700°C temperature at 10\(^{-3}\) and 10\(^{-1}\) s strain rates, see their Fig. 18. Since the test temperature exceeds the critical one, only the athermal part, \( \tau = \tau_0 \gamma^{\frac{1}{3}} \), should be considered. Figure 15 compares the result from \( \tau = 325 \gamma^{\frac{1}{3}} \) MPa with the curves given by these authors for this material. To obtain \( \tau_0 \approx 325 \) MPa, one point (i.e. \( \tau = 250 \) MPa at \( \gamma = 0.27 \)) is fitted to this equation.

**4.8. Empirical use of equation (12)**

One way to use equation (12) is to view all its constitutive parameters as free constants to be estimated empirically. This however, may not be very effective, and it may be better to exploit the physical meaning of various parameters, in order to simplify the required curve fitting.

To this end, it is expedient to first estimate the athermal part, \( \tau_0 = \tau_0 \gamma^{\frac{1}{3}} \), by comparing with experimental data obtained at suitably high temperatures, e.g. 1000–1300K, and at a convenient strain rate, say 1000–5000 s\(^{-1}\). This was illustrated in the preceding subsection, in connection with data of Fig. 12, where...
the three lowest curves directly yield the estimate \( \tau_e = 500 \text{GPa} \).

As a next step, it is useful to construct, from experimental data, the stress difference, \( \tau^* = \tau - \tau_e \), vs the absolute temperature, \( T \), for a constant \( T_0 = 500 \text{K} \). The three lowest curves directly yield the estimate of \( \tau_e \). The flow stress in the circumferential directions, e.g. in forged plates, in both the radial and the low-temperature isothermal stress-strain data. The work reported here has been supported by UCSD’s technique yields such data. In the absence of this capability, one may use the corresponding adiabatic data and again correct for the temperature due to plastic work. Here, however, the flow stress is usually relatively high and more care is required in estimating the increase from the initial temperature. At high strains and high strain rates, and for high temperatures, the entire plastic work may be assumed to have been converted into heat. Since the coefficient of temperature, \( \tau(0) \), in equation (12), is to be estimated, this assumption will not affect the final results, as long as it is consistently used throughout all calculations.

In addition to the high-temperature adiabatic flow-stress data, it is necessary to obtain (or estimate) the low-temperature isothermal stress-strain data. UCSD’s technique yields such data. In the absence of this capability, one may use the corresponding adiabatic data and again correct for the temperature due to plastic work. Here, however, the flow stress is usually relatively high and more care is required in estimating the increase from the initial temperature. At high strains, this temperature increase may be several hundred degrees Celsius. An error in its estimate may significantly affect the results. For this and related reasons, the recovery Hopkinson techniques discussed earlier [1] and in this paper, appear to be indispensable tools.

From the resulting \( \tau^* \) vs \( T \)-curve, the constitutive parameters \( \tau_0, p \) and \( q \) can be estimated, keeping in mind that \( p \geq 2/3 \) and \( q \geq 2 \) for most cases. This leads to an equation similar to equation (3). Then, with an estimate of \( G_\infty \), the reference strain rate, \( \dot{s}_0 \), can be estimated, completing the process. After this, the constitutive parameters may be further ‘tuned’ by comparing with other data.

It is important to note that the parameters \( \tau_e, n \) and \( \dot{\varepsilon} \) are affected by the texture and plastic anisotropy, and that an ‘orientation factor’ affects \( \tau_e \) and \( \dot{\varepsilon} \) differently, because of the exponent \( n \) which is always less than 1/2. Hence, both \( \tau_e \) and \( \dot{\varepsilon} \) need to be estimated empirically for each symmetry direction, e.g. in forged plates, in both the radial and circumferential directions.

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