Stress-wave energy management through material anisotropy

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Abstract

Stress-wave propagation in solids can be controlled through imposing a gradual change of anisotropy in the material elasticity tensor. In this study, a transversely isotropic material is incorporated with a smoothly varying axis of anisotropy. It is shown that if this axis initially coincides with the stress-wave vector, then the energy of the plane waves would closely follow this gradually changing material direction. A fiber-reinforced composite is used to induce the required anisotropy, and to experimentally demonstrate the management of stress-wave energy in a desired trajectory. The material has isotropic mass-density and is considered homogeneous at the scale of the considered wavelengths, even though microscopically it is highly heterogeneous.

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1. Introduction

In recent years, there have been significant efforts to develop techniques to “cloak” an object, making it invisible to electromagnetic waves at a certain wavelength. These cloaks are generally based on using heterogeneous composites with gradually changing electromagnetic properties [1–4]. Schurig et al. [5] experimentally demonstrated EM cloaking at microwave frequencies. Milton et al. [6] studied the properties of conventional elastodynamic equations under curvilinear coordinate transformation and found that in general, equations of motion are not form invariant and therefore the EM cloaking techniques, based on coordinate transformations, are not applicable to elastic stress waves. Cummer and Schurig [7] suggested an acoustic cloaking scheme by noting the equivalence of acoustic and electromagnetic equations in two-dimensional (2D) geometry, using anisotropic mass-density. Chen and Chan [8] suggested a design of 3D acoustic cloak by mapping acoustic equation to DC conductivity equation, again using anisotropic mass-density. Torrent and Sanchez-Dehesa [9] proposed a multilayered composite made of sonic crystals which satisfies anisotropic density requirements suggested by Cummer.

Recently, Norris [10] has shown that if the mechanical stiffness of the material is isotropic, then the inertial density must be finite at the inner surface of the cloak. In fact the inertial anisotropy schemes suggested in the literature are of this kind [7–9]. Cloaking is suggested by numerical modeling in those cases, but fabricating a metamaterial with infinite mass-density is challenging, if not impossible. Norris then shows that perfect cloaking is possible with finite mass through stiffness anisotropy. Pentamode materials (PM) are needed to satisfy the required stiffness anisotropy [6,10]. It can also be used in line with inertial anisotropy (but not infinite mass-density) [10]. Yet, fabrication of such metamaterials is rather challenging. Five of the six eigenvalues of the elasticity tensor vanish in PM and therefore it can be strained in 5 independent modes without producing any stress which causes instability in the material [6,10]. Tehranian et al. [11] found that it is possible to design the microstructure of a material to attain an elasticity tensor which varies smoothly with position; and can be used to guide the energy of stress waves.

Acoustic cloaking can be achieved when stress waves are guided around an object and then re-gathered on the far side of the cloaked region to continue traveling in the same direction so as to make the object seem invisible or protected from the impinging stress waves. A less strict approach may be to seek to minimize the elastodynamic scattering cross section of any object within a
bounded volume by adjusting the local material properties of its surrounding region. The term material properties in this description refers to the effective mass-density and elasticity (or viscoelasticity) tensors that depend on the microstructure at a length-scale that is at least an order of magnitude smaller than the considered window of acoustic wavelengths.

In the present study, we seek to develop a realistic approach to control stress-wave propagation in elastic solids of uniform mass-density but anisotropic elasticity, and demonstrate its feasibility through numerical simulations and experiments at ultrasonic acoustic frequency range. An ideal cloak guides the stress-waves around an obstacle at any angle of incidence and at any frequency. The design suggested in this paper limits the angle of incidence to a direction normal to the boundary surface on which the wave impinges. But, it does that successfully in a relatively wide frequency band. In general, a microstructurally designed cloak cannot operate over the entire frequency spectrum, not only because of theoretical limitations, but also more simply due to the diffraction limit of heterogeneous media. Basic elastodynamic analysis of anisotropic media shows that the longitudinal and shear waves may travel in different directions with different velocities that depend on the angle between the wave vector and the material's principal anisotropy directions [12–14]. However, if the wave vector makes a very small angle with the material's preferred axis of maximum stiffness, then the resulting longitudinal and shear-wave group velocities will make even smaller angles to this direction. In other words, the maximum stiffness direction is also a preferred direction for group velocity and wave-energy flow. Now, if the material direction changes smoothly (with respect to the wavelength) throughout a body, the stress waves will follow the anisotropy direction of the material. Thus, we can control stress waves and redirect them to travel along a pre-designed trajectory.

Musgrave [14] has performed an extensive study on wave propagation in elastic anisotropic media and the corresponding slowness surfaces for different modes. Norris [15] developed a general theory for the propagation and scattering of compact Gaussian shaped envelopes in piecewise homogeneous anisotropic solids. Abrahams and Wickham [16] used an asymptotic method to solve the problem of refraction and propagation of SH waves with relatively large wave-number in a locally transversely isotropic material whose direction of the preferred axis varies as a continuously differentiable function of a single spatial coordinate. Their work is motivated by failure of ray tracing methods in non-destructive testing of austenitic steel welds, which can be modeled as transversely isotropic material with smoothly varying axis of symmetry. Their method can produce a geometrical characterization of the refracted wave. Mazzucato and Rachele [17] studied intersection of slowness surfaces of different modes using microlocal analysis on transversely isotropic materials where the fiber direction may vary smoothly from point to point. Norris and Wickham [18] studied the structure and properties of the ray equation for transversely isotropic materials with moduli which are uniform up to a rotation of the underlying crystallographic base vectors about a common axis. They suggested an algorithm for SH waves that can be used to generate a uniform approximation to the point source field in a smoothly varying medium.

In the present work, we are concerned with longitudinal waves, which, in general, may involve all three displacement components. Given the complexity of the geometry of the problem, we chose to use numerical computation using finite-element method. The moduli of each element are specified based on its spatial location. In Section 2, we seek to provide a basic understanding of the physics of the phenomena using the classical fundamentals of wave propagation in transversely isotropic materials.

Fiber-reinforced composites are used in this work to induce the desired local anisotropy, and yet to create a material that is essentially homogeneous at the scale of the considered wavelengths. One can then change the preferred material axis by controlling the fiber orientation within a sample. In a composite material with a spatially uniform preferred direction, longitudinal waves and shear waves travel in certain directions depending on the angle between the wave vector and the preferred axis (fiber orientation in this case). However, if continuous fibers are oriented in smoothly curved shapes, both longitudinal and shear waves will travel on paths that approximately follow the fibers. The acoustic waves tend to travel close to the direction of the highest stiffness and wave speed. It must be emphasized that unlike wave-guides which exploit geometric surfaces with material discontinuities, the wave redirection considered here is achieved in the absence of discontinuous boundaries. The present three-dimensional solid sample is essentially a homogeneous but highly anisotropic at the scale of the considered wavelengths. Note that the sample is fabricated such that its (varying) elastic modulus tensor at any point is obtained by a pre-designed rotation of its principal directions. It is important to realize that the macroscopic anisotropy of the material guides the acoustic waves, not the microscopic heterogeneity in the composite material. In fact, the scale of the material heterogeneity is what limits the wavelength at which the stress-wave energy can be managed. Fortunately, this range is extremely wide for a fiberglass/epoxy composite material, covering stress waves of up to about 5 MHz frequency. In principle, smaller fibers will allow one to surpass this limit. Applications of acoustic cloaking may include hiding under-water objects such as submarines, protecting a particularly sensitive section of a structure against blast or shock waves, acoustic noise reduction by creating sound-shielding materials, and seismic isolation of civil infrastructure.

2. Elastic waves in anisotropic materials

In anisotropic materials with spatially uniform density, elastic wave propagation is strongly affected by the local principal directions of the material anisotropy. This fact can be used to redirect and manage the elastic stress–wave energy within a material, essentially as is desired. Here, we illustrate this phenomenon using an elastic material which is locally transversely isotropic, but whose local principal plane of isotropy spatially varies from point to point.

With respect to a rectangular Cartesian coordinate system, \( x_i, i = 1, 2, 3 \), the equations of motion are,

\[
\frac{\partial \sigma_{ij}}{\partial x_i} = \rho \frac{\partial^2 u_j}{\partial t^2}.
\]

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where \( \sigma_{ij}, u_i, i,j = 1,2,3 \), and \( \rho \) respectively denote the Cartesian components of the stress tensor, displacement vector, and mass-density [14,19]. The generalized Hooke’s Law is \( \sigma_{ij} = C_{ijkl}u_{kl} \), where comma followed by an index denotes partial differentiation with respect to the corresponding coordinate. Since the elasticity tensor, \( C_{ijkl} \) is spatially variable, Eq. (1) becomes,

\[
\frac{\partial}{\partial x_j} \left( \epsilon_{ij} \frac{\partial u_k}{\partial x_l} \right) = \rho \frac{\partial^2 u_i}{\partial t^2}.
\]

For numerical finite-element simulations, it is convenient to consider a typical finite element, \( x_j, j = 1,2,3 \), denote the local rectangular Cartesian coordinate system, and examine plane wave propagation within this element which is now assumed to be uniform. Then integrate the results into the large-scale finite-element code, LS-DYNA, to solve the corresponding elastic waves propagating through an elastic solid whose principal axes of elasticity tensor may vary from element to element.

We now focus on a transversely isotropic material. Plane wave solution to the equation of motion is in the following form:

\[
u_i = g_i (n_i x_i - V t),
\]

where \( n_i \) are components of the unit vector normal to the wave-plane, moving with speed \( V \), and repeated indices are summed [12]. Substituting this solution into the equation of motion, and noting that the elasticity tensor in the considered element is constant, we obtain,

\[
(C_{ijkl} n_j - \rho V^2 \delta_{ik}) g_k = 0.
\]

For nontrivial solutions, we must have,

\[
det(C_{ijkl} n_j - \rho V^2 \delta_{ik}) = 0.
\]

For an element with transversely isotropic elasticity, we chose the \( x_3 \)-axis normal to the plane of isotropy. The elastic modulus matrix \([C_{ab}]\) is

\[
C_{1111} = C_{11} = D \left( \frac{1}{E_3} \left( \frac{v^2}{E_3} \right) \right), \quad C_{1122} = C_{12} = D \left( \frac{1}{E_3} \left( \frac{v^2}{E_3} + \frac{\nu^2}{E_2} \right) \right),
\]

\[
C_{1133} = C_{13} = D \left( \frac{(1 + \nu) v_3}{E_3} \right), \quad C_{3333} = C_{33} = D \left( 1 - \frac{v^2}{E_2} \right),
\]

\[
C_{2323} = C_{44} = \mu_3, \quad D = \frac{E_3^2 E_2^2}{(1 + \nu)(1 - \nu v_3^2 - 2\nu^2 E_1)}.
\]

These equations involve 5 independent elastic constants. These are: Young moduli \( E \) in the \( x_1 \) and \( x_2 \), and \( E_3 \) in the \( x_3 \) directions; Poisson ratios, \( \nu \) the \( x_1 \) and \( x_2 \), and \( v_3 \) in the \( x_3 \) directions, respectively; and shear modulus, \( \mu_3 \), in the \( x_1, x_3 \) and \( x_2, x_3 \) planes, the shear modulus in the plane of isotropy being \( \mu = E/2(1 + \nu) = (C_{11} - C_{12})/2 \).

Taking \( \mathbf{\hat{n}} = (\sin \theta, \cos \theta) \) for a plane-wave which makes an angle \( \theta \) with the \( x_3 \) axis in the \( x_1x_3 \)-plane, we obtain the characteristic equation,

\[
\left[ C_{44} \cos^2 \theta + \frac{1}{2} (C_{11} - C_{12}) \sin^2 \theta - \rho V^2 \right] \times \left[ C_{44} C_{33} \cos^2 \theta + \left( C_{44}^2 + C_{11} C_{33} - (C_{11} + C_{44})^2 \right) \sin^2 \theta \cos^2 \theta + C_{44} C_{11} \sin^2 \theta - \rho V^2 (C_{11} + C_{44}) \cos^2 \theta - \rho V^2 (C_{11} + C_{44}) \sin^2 \theta + \rho V^2 \right] = 0.
\]

The roots of this equation now yield the inverse (i.e., the slowness) of the velocities of three different plane waves that can travel in a transversely isotropic material, as follows [13]:

1. Quasi-shear mode:

\[
\frac{1}{V_1} = \left( \frac{k}{\omega} \right)_1 = (2\rho)^{1/2} \left[ C_{11} \sin^2 \theta + C_{33} \cos^2 \theta + C_{44} - \sqrt{\left[ (C_{11} - C_{44}) \sin^2 \theta + (C_{44} - C_{33}) \cos^2 \theta \right]^2 + (C_{11} + C_{44}) \sin^2 2\theta} \right]^{1/2}.
\]

2. Pure shear mode:

\[
\frac{1}{V_2} = \left( \frac{k}{\omega} \right)_2 = \left( \frac{\rho}{C_{11} - C_{12} \sin^2 \theta + C_{44} \cos^2 \theta} \right)^{1/2}.
\]
3 Quasi-longitudinal mode:

\[
\frac{1}{V_3} = \left( \frac{k}{\omega} \right)_3 = (2\rho)^{1/2} \left[ C_{11}\sin^2 \theta + C_{33}\cos^2 \theta + C_{44} + \sqrt{[C_{11} - C_{44}] \sin^2 \theta + (C_{44} - C_{33}) \cos^2 \theta} \right]^{1/2}. \tag{10}
\]

As an illustration, Fig. 1 shows the variation of these three quantities for various values of the angle \( \theta \). These curves are drawn for the following elastic constants: \( E = 15.37 \) GPa, \( E_3 = 48.46 \) GPa, \( \nu = 0.34 \), \( \nu_3 = 0.24 \), \( \mu_3 = 5.47 \) GPa. These values are obtained by characterizing a transversely isotropic composite material, as discussed in Section 4. Also, these values are used to produce results discussed in the rest of this section, as well as for all other experimental results that are reported in this paper.

For anisotropic media, the wave vector \( \mathbf{k} = k \mathbf{n} \) (where \( k \) is its magnitude) and the group velocity \( \mathbf{V}_g \) with magnitude \( V_g \) do not always have a common direction, where \( V_g = \frac{\partial \omega}{\partial k} \) [13,20]. Since \( \delta \mathbf{k} \) must always lie in the slowness surface, the group velocity is normal to the slowness surface, as shown in Fig. 1. It can be seen that the direction of the group velocity is different for different modes corresponding to the same angle, \( \theta \). A more detailed discussion of creating such slowness curve is provided in Payton [12], Auld [13] and Mazzucato and Rachele [17]; see however Payton [12] who focuses on transversely isotropic media.

As a simple example, consider a transversely isotropic sample shown in Fig. 2, with the \( x_3 \)-axis making an angle \( \theta = 45^\circ \) with the wave vector \( \mathbf{k} \). In this case, \( \theta \) is uniform throughout the sample. The wave packet travels with a group velocity that makes an angle \( \alpha \) with the \( x_3 \)-direction, as displayed in Fig. 1 for the three waves mentioned above.

![Fig. 1. Slowness curves for a transversely isotropic material.](image1)

![Fig. 2. Deflected acoustic-wave trajectory in anisotropic material.](image2)
Pure shear waves travel almost in the $k$-direction at a velocity of $V_2 = 1755$ m/s. Quasi-longitudinal waves travel at $V_3 = 4070$ m/s, making an angle $\alpha_3 = 11^\circ$, and the quasi-shear waves at $V_1 = 2344$ m/s making an angle of about $\alpha_1 = 65^\circ$ relative to the $x_3$-direction, as shown in Fig. 1. In this case, plane strain is assumed and the displacement in the $y$-direction (vertical direction in Fig. 2) is prescribed over a 20 mm strip as a harmonic sinusoidal wave with 1 MHz frequency. The rest of the boundary is stress-free.

The simulations give $V_3 = 4300$ m/s, and $V_1 = 2322$ m/s, with the corresponding angles $\alpha_3 = 13^\circ$, and $\alpha_1 = 67^\circ$. None of the waves travel in the anisotropy ($x_3$-direction) of the material, as the wave vector does not coincide with the $x_3$-direction. These are plotted in Fig. 3, which shows the effective stress (von Mises) at a specific time after the excitation begins. Red fringes show higher effective stresses (von Mises) while blue shows zero effective stress. The maximum effective stress experienced at each element on the bottom side of the sample is found and plotted (blue dots) below the sample in Fig. 3. Points $B$ and $C$ (marked on the sample) correspond to two local maxima. Line $AB$ shows the direction of propagation of quasi-longitudinal waves and line $AC$ that of quasi-shear waves. Also, the time it takes for waves to travel from point $A$ to $B$ and $C$ is computed based on the 5th sine wave peak. Pure shear waves are traveling slower than the other modes and cannot be seen in Fig. 3, as effective stress in the model is dominated by quasi-longitudinal waves. However, theoretical values show that these waves make an angle $\alpha_2 = 45^\circ$ with $x_3$.

In numerical simulation, wave speeds and deflection angles are calculated based on the von Mises stress which represents distortional energy. This may be one reason for the slight difference between numerical results and the analytical values which are calculated based on slowness curves. Displacement of the nodes can be decomposed in three discussed modes to obtain angles and speeds of the waves.

When the wave vector deviates only slightly from the material’s principal $x_3$-direction, as shown in Fig. 4, then pure shear, quasi-longitudinal, and quasi-shear waves all travel more or less in the $x_3$-direction. Now, if the material anisotropy direction changes slightly, the group velocity will follow the same and changes accordingly. If initially the wave vector coincides with the material’s principal direction and undergoes smooth changes, then the acoustic-wave energy packet would follow a similar path. Thus, it is possible to control the elastic energy path by the judicious design of material anisotropy. This is illustrated, both experimentally and numerically, in the next two sections.

3. Numerical computation

The objective of this section is to design and numerically simulate a possible specimen, which can split, when necessary, and redirect acoustic stress waves around a target object and then re-combine them on the opposite side of the object to produce
waves that have their original spatial distribution, although they may be partially attenuated. We want to ensure that the interior object would be minimally excited by incident waves and thus remain protected and hidden to ultrasonic excitations. We use anisotropic materials to achieve this goal. We focus on transversely isotropic materials and consider cases where the anisotropy direction (referred to as the axis of anisotropy) smoothly and gradually changes within the solid and hence redirects the waves as desired. Except for the direction of the elastic anisotropy, the material appears homogeneous at the scale of the considered wavelengths. In the following subsections we first present a simple model and then analyze a sample which we have fabricated for experimental verification, as discussed in Section 4.

Fig. 4. Slowness curves analysis where k is applied close to x3-axis.

Fig. 5. Cubic sample with circular anisotropy. At each point in a plane normal to the x2-axis, the axis of anisotropy, x3, is normal to the radial line connecting that point to the corresponding point on the edge BB'.

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3.1. Circularly varying axis of anisotropy

A simple model is presented in this section to illustrate acoustic energy management in a transversely isotropic cube, shown in Fig. 5. In this model, the axis of elastic anisotropy at each point is tangent to a circle of a common center. The LS-DYNA finite-element model is a cube with 10 cm long sides as shown in Fig. 5. The local material $x_3$-axis, which coincides with the direction of maximum stiffness, is changing throughout the model. At each element, the $x_3$-axis would be normal to the radial line connecting that element to the specimen’s axis of radial symmetry [21]. A load-free boundary condition is prescribed at all surface nodes; except for those on a 2 cm×2 cm area centered at $T$. These nodes are oscillated with a single 100 kHz sinusoidal pulse (from $t = 1 \mu s$ to $t = 11 \mu s$) along the global $Y$-direction with 10 $\mu$m amplitude, followed by zero $Y$-displacement throughout the rest of the simulation. We are going to look at the effective stress on the mid-plane made by $TRGL$ in Fig. 5.

The maximum stiffness direction, $x_3$, changes smoothly as the wave is propagating in the model. It can be seen from Fig. 6 that acoustic-wave energy is directed along the $x_3$-direction. Initially, both the wave vector $k$ and the $x_3$-direction are in the $Y$-direction, but the group velocity of acoustic waves gradually changes its direction and follows the local material’s axis of anisotropy, $x_3$. The normal displacement of the nodes at the center of cube’s faces, i.e., points $T$, $R$, $G$, $L$, are plotted in Fig. 7; i.e., the $X$-displacement for $R$ and $L$, and the $Y$-displacement for $T$ and $G$. The displacement of node $L$ is much larger than that of the others, as the acoustic wave is redirected towards $L$.

3.2. Stress-wave redirection

In this section, we consider a sample with an interior cavity and examine two transversely isotropic cases to illustrate and contrast the effect of variation of the axis of anisotropy (maximum stiffness direction, the $x_3$-axis) on the acoustic-wave path. The general geometry of the sample is shown in Fig. 8a; see also Fig. 14 for an actual fabricated sample. One sample, (model A), has its

![Fig. 6. (a-d) The von Mises stress in mid-plane, $TRLG$, of the model shown in Fig. 5, at four indicated times after beginning of the simulation.](image-url)
Fig. 7. Normal displacement at the center of four faces of the cube, as identified in Fig. 5.

Fig. 8. Orientation of the elastic anisotropy axis, the $x_3$-axis, in: (a) the baseline model with uniformly straight anisotropy axis, and (b) with an anisotropy axis that follow the indicated curved path around the central cavity.

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Fig. 9. (a) The von Mises stress contours from numerical simulation of model A, plotted for a selected time sequence. (b) The von Mises stress contours from numerical simulation of model B, plotted for a selected time sequence.
$x_3$-axis parallel to the $X$-direction at each point within the sample, Fig. 8a, while in the other sample, (model B), the direction of the $x_3$-axis follows a smoothly curving path around the central cavity, Fig. 8b. The density and degree of anisotropy ($E_3/E_1$) are constant throughout both models. That is, any two elements taken from different parts of each model have identical density and elasticity tensors in their corresponding local principal anisotropy coordinate system. Only the axes $x_1$ and $x_3$ change with respect to the global coordinate system for model B, but not for model A, while their $x_2$-direction remains uniform in both cases.

Model A, (the sample with a unidirectional transverse isotropy, Fig. 8a), is simulated as a baseline to contrast the results with the stress-wave path in model B of Fig. 8b. The local elastic constants at each point are the same as those in Section 3.1, Fig. 5. Both models are 11 cm long in the $X$-direction, and 5 cm wide in the $Y$-direction.

Model B has the same geometry, same homogeneous mass-density, and same elastic moduli in local material coordinates as model A, but with different local material axes $x_1$ and $x_3$ while $x_2$ remains constant throughout the model (Fig. 8b). Solid lines in Fig. 8b show how the $x_3$-axis varies in the model in order to conform to the geometry of central cavity. The $x_1$-axis is chosen such as to render the 123 coordinate system right-handed and orthogonal.

Each of the models is subjected to a single 1 MHz sinusoidal pulse of 100 N force in the $X$-direction over each node on a 1 cm strip centered at point M, of one face of the sample (left face in Fig. 8a and b). The sinusoidal load is applied from $t = 2 \mu s$ to $t = 3 \mu s$, and after that all nodes are load-free. The rest of the boundary, including the central cavity is stress-free in both cases. In order to solve the problem in plain strain, out-of-plane degrees of freedom (the $Z$-displacement) are constrained for all the solid elements.

Finite-element calculation is performed using LS-DYNA. The effective stress (von Mises) is computed in each element at each time-increment. The von Mises stress represents distortional energy and can be used to study the acoustic energy trajectory in the medium. The results are discussed below, starting with model A.

3.2.1. Model A

The results for this baseline case are illustrated in Fig. 9a. Red fringes show higher effective stresses while blue shows zero effective stress. The considered loading induces plane waves propagating in the direction of elastic anisotropy, i.e., at each point,

![Diagram](image)

**Fig. 10.** (a) Geometry of the model, (b, c) Numerical results in central excitation and comparison with experimental data. The light and heavy solid lines are the time maximum of the axial strain as calculated in numerical simulation for models A and B. The solid squares and triangles are experimentally measured voltage signals by ultrasonic transducers (see Section 4). In (b) the numerical simulation results and experimental data are taken at the end surface of the sample, while (c) shows these quantities at cross section $A$–$A$. In each of the two graphs, only the peak experimental and numerical values for sample B are normalized to have the same geometric magnitude. The normalization factors for graphs (b) and (c) are different. The experimental and numerical profiles for model B are in close agreement, but they are substantially different than those in model A.

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the wave vector, $k$, coincides with the corresponding $x_3$-axis. Slowness curves in Section 2 with $\theta = 0$ can be used to predict the direction of the group velocity. All three modes of the plane wave travel in the $x_3$-direction, as confirmed by Fig. 9a. As the plane wave encounters the stress-free surface of the central cavity, it is reflected off that surface, scattering throughout the model as can be seen in Fig. 9a(1–6).

3.2.2. Model B

Here again, the wave vector is initially in the $X$-direction that coincides with the local material $x_3$-axis. As the acoustic wave propagates in model B, it follows the direction of the $x_3$-axis that is changing, travels around the cavity, and finally returns to its initial direction. As suggested by the discussion on slowness curves in Section 2, the acoustic-wave packet should follow the $x_3$-direction. Fig. 9b shows selected snapshots of the von Mises stress-contour plots for model B. Fig. 9b(1, 2) is essentially the same as those for the baseline model A; see Fig. 9a(1, 2), since the anisotropy of the path traveled by the waves is similar. Since for model B the $x_3$-axis conforms to the geometry of the boundary of the central cavity, the wave path similarly changes and is redirected around the cavity. There is a small sudden change in $x_3$-direction at the transition from linear to curvilinear anisotropy along the sample. Fig. 9b(3–5) shows how most of the energy of the acoustic wave has followed the $x_3$-axis of the model. There are small reflections off the cavity surface that may be the result of small sudden changes in $x_3$-direction. However, the energy of the reflected waves is small compared to the bulk of the energy that travels around the cavity. Fig. 9b(6, 7) shows how each of the two wave packets has traveled, one above and the other below the central cavity, finally join to form a single packet of the plane wave that then travels along the $x_3$- or $k$-direction, with most of the energy being concentrated at its center. Also, the sharp peaks in Fig. 10c show that most of the energy of the acoustic waves has traveled very close to the opening surface. (Fig. 9b(4) is the corresponding state.)

In Fig. 10 we have plotted the maximum values of the $\varepsilon_{XX}$-strain attained over the considered time interval, at the end section of model A, in Fig. 10b, and for section A–A (mid-section of the model), in c. As can be seen in Fig. 9a, most of the energy of the acoustic waves is scattered in the left half of the sample in model A. The light solid lines in Fig. 10b and c represent the numerical results for the baseline model A. The heavy solid lines represent the simulation results for model B. In comparison with similar graphs for model A, the wave redirection and re-formation is clearly demonstrated. Furthermore, we have normalized the signal measured experimentally using ultrasonic transducers (see Section 4 for details), in such a way that the peak of the experimental and the numerical profiles are equal. The experimental signals are shown by solid squares in the Fig. 10b and c. After peak calibration, the experimental and numerical profiles are in good agreement.

Fig. 11. (a) Geometry of the model, (b, c) Numerical results in off-center excitation. In (b) and (c), the solid lines are the results of numerical simulations; while the solid squares are experimentally measured signals (see Section 4). In each of the two graphs, only the peak experimental and numerical values are normalized to have the same geometric magnitude. The normalization factors are different in graphs (b) and (c).
Fig. 10b and c shows that the strain peak for model B is about 10 times greater than for model A at the A–A-cross section; and more than three times greater on the surface of the model. Furthermore, comparing Fig. 9a and b it is seen that, in model B, the amount of energy transferred from the left-half of the model to the right-half is considerably greater than that of the baseline model. In fact, the small disturbance observed on the surface of the baseline model might be the result of surface waves and strongly depends on the boundary conditions, whereas in model B, the acoustic wave is guided in the designed direction and therefore most of its energy is concentrated in its trajectory. Thus, the transfer of the ultrasonic stress-waves is managed in this example by controlling the material anisotropy only and not the mass-density.

3.3. Off-center excitation

It is of interest to examine a case where the excitation in model B is applied off-center on its left face. Fig. 11a shows the position of the nodes on which the sinusoidal excitation is imposed. The center of the excitation is 6.3 mm off center of the model’s left face. The maximum values of the $\varepsilon_{XX}$ strain over the considered time interval are plotted in Fig. 11b and c at the end surface and mid-section of the model, respectively. The peak in maximum axial strain occurs at about 6.4 mm off center, on the right face of the sample, very close to the eccentricity of the incident input pulse. The solid squares in Fig. 11 correspond to the normalized measured data discussed in Section 4.

Fig. 12 shows how the effective stress is distributed in the model at a selected time sequence. Essentially, the same phenomenon that occurred in the centrally excited case is observed in the present case. The ultrasonic wave packet is guided around the central cavity of model B, and is delivered on the right-face over almost the same exact location as the corresponding location of the incident pulse.

4. Acoustic waves in anisotropic composites

We have fabricated transversely isotropic fiber-reinforced composites, using fiberglass-epoxy prepregs, to produce a macroscopically homogeneous solid with desired direction of the principal axis of anisotropy. This direction, which corresponds to the fiber orientation, is associated with the direction of the highest overall stiffness, and can be controlled by adjusting the
orientation of the prepreg layups. In this manner we have designed and fabricated samples that clearly show how the overall anisotropy can be used to guide stress waves within a material that, at the scale of the wavelength, is essentially homogeneous; even though microscopically (i.e., at smaller length-scales) it is highly heterogeneous. The main objective of this paper is to show this phenomenon experimentally, and qualitatively confirm the results by numerical simulations.

Fig. 13 shows the micrograph of our fiberglass-epoxy composite. As is seen, the diameter of the glass fibers is less than 20 μm, having less than 10 μm spacing. The ultrasonic waves that we have used are in the low MHz frequency range. The minimum measured longitudinal and shear-wave speeds in this composite are around 3000 and 1500 m/s respectively. With these speeds, the corresponding wavelengths are about 3 and 1.5 mm for 1 MHz ultrasonic waves. Thus, for this kind of ultrasonic waves, the material is effectively homogeneous, yet highly anisotropic. In fact, in a fiberglass-epoxy composite, acoustic-wave redirection is possible for stress waves up to about 5 MHz frequency.

Based on available data, we have assumed Poisson ratios of $\nu_3 = 0.24$ and $\nu = 0.34$ for the fiberglass-epoxy composite, and calculated the other elastic moduli from the measured wave velocities: two longitudinal velocities, one in the $x_3$ and the other normal to this direction ($V_1 = 5310$ and $V_2 = 3150$ m/s respectively); and one shear velocity in the $x_1$, $x_3$-plane with the $x_1$-polarization ($V_3 = 1735$ m/s). Samples of different lengths were used for velocity measurements. These measurements
verified that this composite behaves essentially as a transversely isotropic material with Young’s moduli, $E_2 = 48.46$ GPa and $E = 15.37$ GPa, and the shear modulus, $\mu_3 = 5.47$ GPa. The basic equations used to obtain these constants, are,

$$C_{33} = \rho V_{1}^{2}, \quad C_{11} = \rho V_{2}^{2}, \quad C_{44} = \rho V_{3}^{2},$$

where density $\rho$ is measured to be 1820 kg/m$^3$, and $C_{11}$, $C_{33}$, and $C_{44}$ are given in terms of the elastic moduli in Eq. (6).

4.1. Sample fabrication

Unidirectional composite plates of various thicknesses but a common glass volume fraction (49%) were fabricated and used to characterize the overall properties of the material. To experimentally verify wave redirection through elastic anisotropy that is suggested by the numerical simulations, we designed and fabricated a locally unidirectional fiber-reinforced composite sample of the geometry used for simulations; see Section 3, Fig. 8b. An aluminum mold of the desired geometry was first fabricated (Fig. 14a). Then unidirectional glass/epoxy prepreg sheets of suitable lengths were stacked on the mold in a pre-calculated sequence to ensure that the fiber content of the resulting composite sample would be essentially uniform throughout the sample; thus the fiberglass areal density would be uniform on any cross section taken normal to the fibers (i.e., normal to the sample’s long axis). Since the smallest thickness of the sample is half its greatest thickness, every other prepreg sheet was continuous while every other one consisted of two equal-length sheets, cut to a size to ensure the uniform glass-fiber density. Table 1 gives the length of 127 prepreg sheets in the layup. We used prepreg layers consisting of thin sheets of S-2 glass fibers impregnated with uniformly distributed epoxy matrix and then partially cured. They were then cut, placed in the mold, and processed as its final stage of curing. Before placement of the prepreg layers, the mold was covered with Teflon sheets. The mold with prepreg layers is then vacuum-bagged and pumped down to 10 mm Hg pressure. The layup was then placed in a 125ton Wabash laminating hot press, and subjected to a pressure of 0.35 MPa while curing. The composite panels were processed in accordance with the manufacturer’s recommendations. The curing was done using a 3 °C/min heating rate up to a final temperature of 121 °C. Then the sample was kept at this temperature for 1.5 h. The setup remained in the hot press under pressure and cooled to room temperature [22]. The cured composite was taken out of the mold and the surfaces were machined to achieve smooth faces. The panel was then cut into top and bottom pieces which were then glued together with AeroMarine 300/21 Cycoaliphatic epoxy. The fiber volume fraction of the composite material was measured to be 49%, by ignition loss testing, in accordance with ASTM D 2584-02.

Fig. 14a shows the mold, and b the resulting composite after it has been hot-pressed, cooled, trimmed, and its surfaces machined. Then the composite was sectioned into three parts, two of these were glued together with epoxy to obtain sample B in Fig. 14c, and the third part was cut and glued to obtain sample C of Fig. 14d. The final sample B consists of two end segments with uniform preferred $x_3$-direction and middle sections with a gradually changing elastic anisotropy axis (the $x_2$-axis). This sample was used to investigate acoustic-wave trajectories. In addition, sample C was used to measure the acoustic waves as they cross a plane normal to the fibers, half-way through the length of the sample. In this study, the geometry of the sample is particularly selected to illustrate the phenomenon of guiding elastic waves (with an angle of incidence normal to the sample boundary) around a region within an anisotropic (yet homogeneous) solid. The geometry of the curved molds is calculated and the molds are designed to provide a smooth change in the orientation of the anisotropy directions of the resulting samples, in order to accomplish this. As demonstrated in Fig. 15, the mold surfaces consist of circular arcs and straight-line segments. This design can

Table 1
Length of prepreg sheets in layups in centimeters.

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be machined with high precession, and is suitable for the hand layup composite fabrication. In general, it is possible to vary the shape of the internal (cloaked) region to a great extent, as long as the composite volume can be filled with a stack of properly cut prepreg layers without creating voids. The current design is chosen for geometric simplicity of alternating cut and uncut prepreg stacks. External surfaces of the molds are flat for ease of use with the hot press processing technique.

4.2. Experiment

Olympus Panametrics-NDT ultrasonic transducers were used to send and receive the ultrasound-frequency stress waves through the samples. These transducers are based on piezoelectric or ferroelectric materials that can convert electrical signals to mechanical signals and vice versa. The transducers create a normal or tangential surface displacement, producing mostly longitudinal or polarized shear waves in the sample. The input signal is produced using a Matec TB1000 tone-burst generator card. The tone can be adjusted with 100 Hz frequency resolution. The line is split and one line modulates the input transducer, while the other is reduced and sent to a digitizer for measurement. The output transducer signal is also sent to the digitizer. Olympus V103 transducers are optimized to modulate pulses around 1 MHz frequency content. Low viscosity oil was applied to the interfaces of the transducers and the sample to facilitate the transmission of acoustic waves. The received signals were measured on a square reference-grid with 6.3 mm spacing to establish the propagation path of the stress waves within the composite; see Fig. 14c, d. The receiving transducer was placed on the grid points and affixed to the sample. The incidence transducer was placed at a number of fixed points as described below. The amplitude of the received electrical signals was measured and compared to other measurements for the same placement of the incidence transducer. The electrical voltage pulse created in the transducer due to the mechanical oscillation at the interface was measured. Since the system is fully linear, the maximum voltage is proportional to the maximum particle velocity and maximum displacement. Comparison of the amplitude of the received signals demonstrates how the energy of the stress wave is transmitted to the opposite surface of the sample. The graphs shown in what follows are therefore drawn with arbitrary units.

4.2.1. Central excitation

In the first set of experiments, the actuating transducer is placed at point M (Fig. 14c, d) on sample B or C. In each test, the receiver is placed at one of the grid points shown in Fig. 14c, d, and the received signal is recorded. For each received electrical signal, the maximum instantaneous amplitude is found and normalized with respect to the amplitude of the incident pulse. The results are summarized in Fig. 16.

Experimental results in Fig. 16a demonstrate that the measured transmitted signal is maximum at the center of the opposite sample-face, in a small neighborhood of point P, although a straight line from this point to the actuating transducer passes through the central cavity of the sample. To better understand the wave propagation trajectory in the sample that produced the peak signal at P, we measured the transmitted signal on the opposite faces of sample C.

As Fig. 16b shows, the maximum acoustic energy is measured in sample C around points denoted by R and R′, very close to the surface of the central cavity. Therefore, the stress wave travels essentially along a path defined by the axis of elastic anisotropy (maximum stiffness), in this transversely isotropic material. At the beginning of the path, the ultrasonic pulse travels along the fibers (which define the x3-direction) in the locally unidirectional composite material. As the fiber orientation (and hence the x3-direction) gradually changes, the stress pulse is redirected to travel along the material’s preferred x3-direction. The wave energy splits into two parts near the central cavity and travels along the surface of the cavity that coincides with the curved direction of highest stiffness (the x3-direction). Traveling acoustic waves on the two sides of the opening then join together and finally follow...
the constant direction of anisotropy at the end, giving rise to a single peak at point \( P \) of sample B in Fig. 16a, and two peaks in Fig. 16b (at points denoted by \( R \) and \( R' \)) in sample C.

4.2.2. Off-center excitation

In the second set of experiments, the actuating transducer was shifted 6.3 mm from the center of the surface away from the hollow part (point \( T \)); see Fig. 17. The maximum amplitudes of the received waves are normalized similar to the previous tests. The result can be seen in Fig. 17. The peak response is measured at 6.3 mm from the middle, exactly at the same point as excitation happens but on the opposite face (Point \( H \)), demonstrating that the acoustic-wave trajectory in the material falls in the direction of highest stiffness.

The results of central and off-center excitation experiments shown here suggest that wherever on one face, normal to the axis of greatest stiffness of sample B, an acoustic wave is modulated, it will be mainly received at the same position on its opposing face. In other words, the curved free surface of the central cavity only minimally scatters the acoustic stress waves. The anisotropy of the

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composite redirects the incoming acoustic stress waves around the cavity and directs them towards the opposite face. Thus, controlling anisotropy of the material allows for management of acoustic stress-wave energy.

4.3. Fabrication and experiment on baseline model

Two baseline samples were fabricated by mimicking the geometry and anisotropy configuration of baseline model A proposed in Section 3.2 (Fig. 8a). Unidirectional composite sample is made by hot-pressing a prepreg layup in flat molds. The cured product is carved with a CNC mill to attain the same internal geometry of the cavity in model B as shown Fig. 14a, and then cut into top and bottom pieces which were glued together to reproduce the same geometry as samples in Fig. 14b, c. These samples make it

Fig. 17. Off-center excitation: maximum instantaneous amplitude of the received electrical signal, normalized with respect to the amplitude of the incident pulse in (a) sample B, and (b) sample C. The plots are drawn by interpolating the experimental data measured on a square reference-grid with 6.3 mm spacing. Point T is 6.3 mm away from the center of the lower face M. Note that different scales are used for the left- and right-plot.
possible to observe the geometrical effect of cavity on wave propagation in a unidirectional anisotropic material. The ultrasonic experiments are performed on these samples and the results are shown with solid triangles in Fig. 10b, c. These data may be compared with experimental results on Section 4.2.1 (solid squares in Fig. 10b, c). It is seen that in the baseline model, the inside cavity scatters the incident stress-waves and only small disturbance is measured on the end faces; while the experiments on the sample with designed anisotropy shows that the incident stress-wave is guided around the cavity, and relatively large axial strains are measured on the faces.

5. Summary

In this work we have shown numerically and experimentally that managing the energy of stress-waves is possible by designing the preferred axis of a transversely isotropic material to vary smoothly with location. This controlled anisotropy is induced by a heterogeneous microstructure, however, when the wavelength of stress waves is long enough, the overall response is an effective homogeneous one. In transversely isotropic materials, when the wave vector deviates only slightly from the axis of maximum stiffness, almost all the energy of different modes of plane wave will travel along this axis. We have designed a specimen in which the anisotropy direction changes smoothly in order to split and redirect stress waves around a target object and then re-combine them on the opposite side of the object (referred to as “acoustic cloaking” in the literature). It is numerically shown and experimentally demonstrated that the acoustic-wave energy packet would follow a similar gradual change as the axis of anisotropy.

A fiber-reinforced composite is fabricated to produce a macroscopically homogeneous solid, with fiber orientation creating the axis of maximum stiffness in the locally transversely isotropic material. We can design the heterogeneous microstructure of the specimen by adjusting the orientation of the prepreg layups. The purpose of the design is to redirect all the stress-waves with a given wave-vector direction around a cavity inside the specimen and deliver it on the opposite side of the cavity. It is shown that the cavity which may be filled with any desired material remains hidden to ultrasonic measurement tools and scatters stress waves minimally. The immediate application of such technology is to protect sensitive objects or facilities against undesirable acoustic disturbances.

Acknowledgments

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References


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